Improved Orbit Estimation Using GPS Measurements for Conjunction Analysis

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Biography

Alana Muldoon received dual degrees in Mathematics and Computer Science from the University of California at Santa Cruz in 2007. She is currently a Masters student in the Computer Engineering department, where she is pursuing research in orbital debris mitigation.

Gabriel Elkaim received his B.S. degree in Mechanical/Aerospace Engineering from Princeton University, in 1990, and both M.S. and Ph.D. degrees from Stanford University, in Aeronautics and Astronautics, in 1995 and 2002 respectively. In 2003, he joined the faculty of the Computer Engineering department, at the Jack Baskin School of Engineering, University of California, Santa Cruz, as an Assistant Professor. His research interests include control systems, sensor fusion, GPS, system identification and autonomous vehicle systems. His research focuses on intelligent autonomous vehicles, with an emphasis on robust guidance, navigation and control strategies.

Abstract

Orbital debris is a significant hazard for satellite and manned space flight. With upwards of 12,000 pieces of debris moving at very high speeds, satellites move out of the way rather than risk a catastrophic collision. Analysis of collision probability between space objects is often accomplished by using NORAD's Two-Line Elements (TLEs). TLEs are time stamped orbit position data, updated periodically, that include neither error estimates nor any guarantee of their accuracy. This uncertainty burdens satellite owner/operators, forcing them to take unnecessary actions to avoid other space objects, using up precious fuel and resources, and shortening satellite lifespan. In this paper we compare TLEs to GPS precise ephemerides to find systematic rotational biases in the TLEs, leading to an improvement in positional accuracy of the GPS TLEs. These rotational biases are remarkably stable, showing improved orbit prediction as far as two years out. The remaining error in the TLEs shows structure, and can be predicted. Future efforts will apply this analysis to LEO objects in order to asses the utility of these improvements to the TLEs.

1. Introduction

On January 11, 2007, the Chinese used an anti-satellite weapon to destroy their own weather satellite, the Fengyun FY-1C, in order to demonstrate that they possessed the technology to do so [1]. The target satellite, in a polar orbit at 865 km altitude, was destroyed with a direct kinetic kill vehicle with a closure rate of over 8 km/s. Following this test, the amount of space debris tracked increased by over 1335 pieces larger than 10 cm across. Due to the orbit altitude, the vast majority of these pieces of the Fengyun will not deorbit for well over a decade [2]. The estimate for the smaller size debris is over 35000 of 1cm across, and over a million pieces 1mm across. Note that current technology can only track objects 10 cm or larger in cross section, and the Chinese ASAT test increased the number of objects tracked by over 15%.

In 1996, a French satellite named Cerise was damaged by orbital debris impact, causing a portion of the gravity-gradient stabilization boom to be torn off and severely degrading the satellite [2]. Currently, space shuttle trajectories are carefully planned to avoid known space debris conjunctions, and clearing maneuvers have been performed on several recent missions. Indeed, parts of the International Space Station are armored to prevent loss of life through orbital debris impact, and the station will perform a collision avoidance maneuver if another object is predicted to come within 25 km of the station.

As these examples demonstrate, space debris is an ongoing and difficult problem to address. Space debris, or space junk, is composed of defunct satellites, rocket parts, casings, paint flecks, garbage, gloves, cameras, and other sundry parts of once very expensive technology. Estimates from the US Space Surveillance Network place the number of objects in orbit at over 12351, with slightly over 3101 listed as payloads, though many of them are defunct. Again, given the ability to only track objects larger than 10 cm across, the actual number of objects in orbit exceeds the tens of millions, though most of these are quite small. The number of conjunctions (or times when two objects will pass closer than some minimum distance) rises dramatically as the number of objects increases.

Due to the extremely high velocities involved and the high probability of head-on collision, even small debris can pose significant danger to spacecraft and astronauts. Unlike collisions in the air, the pieces do not then fall back to earth after colliding but rather continue in roughly the same orbits, spreading out and colliding with other objects. This in turn creates more debris, which leads to further collisions, and so on. This chain of dominoes is known as the "Kessler Effect." [3][4][5][6][7]

While debris in low earth orbit eventually de-orbits due to atmospheric drag, anything in higher orbits will take many decades or centuries to reenter the atmosphere. Proposals for any kind of orbital sweeper remain too costly and far fetched to be effective, and there is currently neither clear legal liability nor established venues to seek redress. The problems caused by orbital debris will not go away on their own, and without any further intervention, will continue to worsen. Near earth space is on the way to becoming another example of the "tragedy of the commons."

It is becoming increasingly important to know the position of satellites, space debris, and other tracked objects with great precision in order for satellite operators to avoid them. If a satellite were to collide with another satellite or space debris, the collision would be catastrophic. The North American Aerospace Defense (NORAD) currently distributes time-stamped orbital data of over 12,000 space objects that are currently being tracked. This orbital data is provided in the form of Two-Line Elements (TLEs), which currently contain no information regarding accuracy or error covariance.

In order to guarantee that a satellite does not collide with another object, large error bounds are placed on the positional accuracy of the TLEs. The larger these error bounds, the more likely it is that a collision will be predicted. The Space Traffic Management Final Report found a direct correlation between the size of the conjunction box and the number of predicted collisions. As these increased, so did their associated cost, both in terms of fuel and the satellite's overall lifespan. [8].

As shown by Figure 1, as the accuracy of the debris location decreases, so the cost of avoiding that debris increases. With a larger error in position comes a larger number of potential conjunctions to be avoided.

Figure 1 demonstrates that better positional accuracy in the TLEs will result in fewer conjunctions and reduced cost. Availability of better position data would also result in better collision prediction in the future, as the orbital propagation diverges in time. This would be a highly advantageous improvement, for the sooner a possible collision is detected, the sooner an avoidance maneuver can be initiated to avoid it. With advanced warning of a possible collision, less fuel is needed, as a small adjustment days or weeks ahead can move a satellite as far as a larger thrust can on shorter notice. The International Space Station (ISS) had to make such a last minute maneuver in August of 2008, using up fuel that was needed for a post-separation test and thus limiting the ISS' future missions [9].

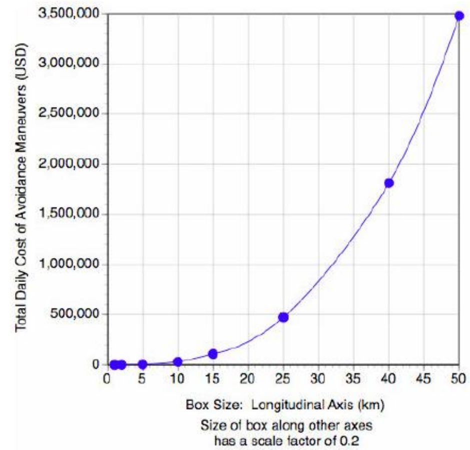

Figure 1: Growing cost with conjunction box size, curtesy of ISU [8]

In order to increase the positional accuracy of the TLEs, we compare propagated TLEs to GPS precise ephemerides to find a systematic rotational bias between the two coordinate systems. We show that if this rotational bias is known, it significantly reduces the errors in the TLEs at the time the bias was found. We also show that the positional accuracy can be improved even two years later by applying the same rotational bias, and that there is a rotational correction that varies from TLE to TLE that does not appear to be random.

2. Comparing to Truth Data

TLEs are represented in the true equator, mean equinox (TEME) coordinate system, which is a product of Simplified General Perturbations SGP4 model developed in the late 1960's [10]. In order to minimize the amount of error that may be introduced unknowingly, no initial coordinate changes are made to the TLEs. Corrections for nutation, precession, or polar motion would only change the positions by meters when errors in the TLEs are currently measured in kilometers. In this paper, we restrict our analysis to just the GPS satellites as they represent the best possible publicly available, accurate data that we can compare to the TLEs. GPS precise ephemerides are available through the National Geospatial-Intelligence Agency online [11]; TLEs are accessed via Space-track [12]. This comparison makes it very straightforward to find a systematic bias between the two frames, as the precise ephemerides have very high accuracy. TLEs are propagated using the code found in Revisiting Spacetrack Report #3 [10].

GPS ephemerides are an excellent example of very accurate "truth data" against which to compare TLEs. Kelso does a very thorough initial comparison of GPS precise ephemerides to TLE data using the STK software to make coordinate transforms [13]. The methods in this paper, however, do not use the STK software. We transform the GPS precise ephemeris into the corresponding TEME coordinate frame for comparison. The precise ephemerides are supplied in WGS-84 coordinate frame (the most current and accurate geocentric frame that rotates with the earth). For our purposes, we have assumed this frame is equivalent to Earth Centered Earth Fixed (ECEF–a generic geocentric frame that rotates with the the earth) as the two frames differ on the centimeter level [14]. Further, we assume that WGS-84 and ECEF are equal to the Pseudo Earth Fixed (PEF) frame (another geocentric frame that includes polar motion), as our calculations showed that polar motion has at most submeter effects. We convert the GPS ephemerides directly to the TEME coordinate frame using ROT3 (-GMST82), which is a conversion from PEF to an inertial frame where motions of satellites is more intuitive [15]. This conversion represents a rotation around the Z axis based on Greenwich Mean Sidereal Time and represents a coordinate rotation from PEF to TEME, or in our case WGS-84 to TEME. We compare this computed TEME coordinate frame (it differs between the actual TEME frame in which the TLEs are represented) to the TEME frame as it is realized in the TLEs. Our aim is to find the best possible conversion from TEME to a frame that is better documented.

For our comparison of the GPS precise ephemerides and their respective TLEs, we considered what the best possible data would be to find a rotational bias. Using the position as reported by the TLEs directly yields insufficient data points for a least squares fit. Instead, we propagate the orbit based on the TLEs and use this data to compute the ideal quaternion from TLE positions in the TEME to GPS positions converted to TEME.

Three types of data are analyzed: using one week of backwards propagated data from a single TLE, using one week of forward propagated data from a single TLE, and using a combination of one week of forward propagated data and one week of backwards propagated data from a single TLE. Only one TLE was looked at for simplicity. Analyzing multiple TLEs at once will be the subject of future work. The TLEs for this analysis were from GPS satellites that had no outages for the entirety of 2006, and whose TLEs were consistent over time. An extra two or three weeks of data is propagated forward (to fill an entire month) in each case to validate the information gathered from the previous data effects data in the future; the ultimate goal is to propagate orbits to perform conjunction analysis as far out into the future as possible. As GPS precise ephemerides are made available in 15 minute increments, we generated TLE position data to match this same format.

The propagated GPS TLE data is then taken and compared directly to their precise ephemerides. We treat the two versions of the TEME frame as two separate coordinate systems and attempt to find the best conversion between the two frames. The best coordinate conversion is found using Horn's closed-form solution to find absolute orientation using quaternion's[16]. Horn's method can be used to calculate the best rotation that minimizes the RMS error between two sets of corresponding points.

To simplify matters, we will use the following abbreviations:

$$T_{TLE} = \text{The TEME frame as represented by the TLE's}$$

$$T_{PE} = \text{The TEME frame as represented in current literature}$$

$$\text{Quaternion} = \begin{bmatrix} q_0 \\ \hline q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

The quaternion found represents an amount of rotation q_0 about an axis of rotation (q_1, q_2, q_3). A rotation matrix from one coordinate frame to the other can be easily calculated from this quaternion.

$$R_{TTLE}^{TPE} = \begin{pmatrix} \left(q_0^2 + q_x^2 - q_y^2 - q_z^2\right) & 2\left(q_x q_y - q_0 q_z\right) & 2\left(q_x q_z + q_0 q_y\right) \\ 2\left(q_y q_x + q_0 q_z\right) & \left(q_0^2 - q_x^2 + q_y^2 - q_z^2\right) & 2\left(q_y q_z - q_0 q_x\right) \\ 2\left(q_z q_x - q_0 q_y\right) & 2\left(q_z q_y + q_0 q_x\right) & \left(q_0^2 - q_x^2 - q_y^2 + q_z^2\right) \end{pmatrix}$$

Using this method, the best quaternion representing the rotation from the TLE's TEME frame to the TEME frame as understood using GPS precise ephemerides is found separately for each satellite. The data used for the purposes of this paper is from January of 2006, concentrating on GPS satellites 1, 2, and 22, as these had no outages for 2006 and represent 3 of the 6 GPS satellite inclination bands.

Note that for very small angles (assuming that $\sin\gamma \simeq \gamma$), the rotation matrix for a $[3 - 2 - 1]$ euler angle set can be written as:

$$R = \begin{bmatrix} 1 & -\varphi & \theta \\ \varphi & 1 & -\phi \\ -\theta & \phi & 1 \end{bmatrix}$$

where φ is yaw, θ is pitch, and ϕ is roll. This small angle rotation matrix will be used to demonstrate what biases show up systematically in the TLEs.

2.1. A Per TLE Comparison

Using the method described above, a consistent bias over all GPS satellites from January of 2006 was found:

$$R(Jan06)_{T_{TLE}}^{T_{PE}} = \begin{pmatrix} 1 & .0002 & 0 \\ -.0002 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The quaternion used to generate this matrix was found using a single TLE from January 7th of 2006 with one week of backwards propagated data and one week of forward propagated data for all of the GPS constellation simultaneously. That is, the least squares fit to the data of all of the GPS ephemerides to the TLE orbit propagation was found; less and/or different data could be used to obtain the same result, and we are simply taking advantage of the amount of available GPS precise ephemerides.

This rotation matrix represents a rotation around the z-axis, the best rotation from T_{TLE} to T_{PE}, and is consistent among all best rotations for all GPS satellites. This corresponds to a yaw between the two coordinate frames of -0.2 mrad, which although small, still contributes to large (especially along track) errors. This result was insensitive to what type of propagated data was used: forward propagated data, backwards propagated data, or a combination of the two. Each resulted in almost the exact same quaternion, which is consistent with a systematic bias in the TEME reference frame as referenced to the ECEF frame. For consistency, all calculations in this paper use a combination of one week of backwards and forward propagated data each.

This rotation around the z axis varied by about .0001, or stated another way, the yaw between the two coordinate systems slipped by 0.1 mrad over 12 months later, and differed by .0003 (0.3 mrad) after two years. The best consistent rotation from T_{TLE} to T_{PE} in January of 2008 is:

$$R(Jan08)_{T_{TLE}}^{T_{PE}} = \begin{pmatrix} 1 & .0005 & 0 \\ -.0005 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

again, corresponding to a slipping in yaw of 0.5 mrad between the two coordinate frames. These values stayed consistent for every GPS TLE over time and give the best possible rotation from T_{TLE} to T_{PE} for GPS satellites in January of 2006. An insufficient number of TLEs have been tested to prove that the same amount of change is consistent across all TLEs, namely TLEs other than those representing GPS TLEs as very little accurate data from other satellites is easily available.

Figures 2,3, and 4 represent the error in the RIC (radial, in-plane, and cross-track) coordinate frame for three different GPS satellites including the constant rotational bias.

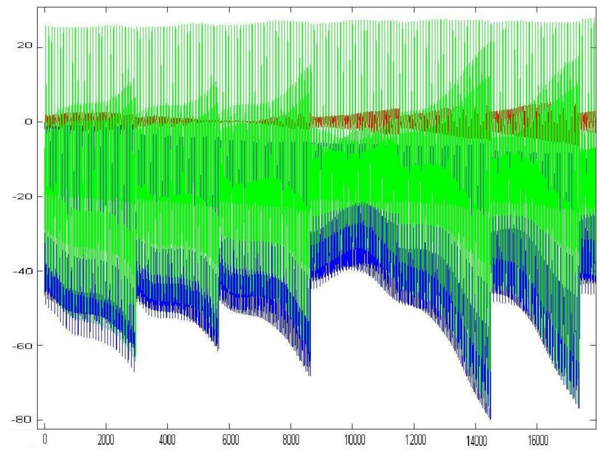

Figure 2: PRN1 error in TEME with R(Jan06)

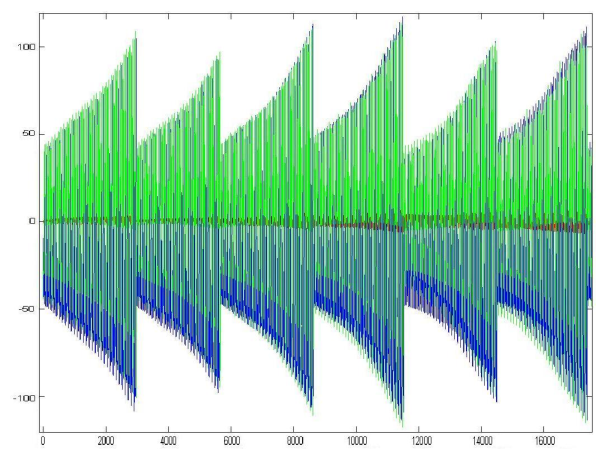

Figure 3: PRN2 error in TEME with R(Jan06)

They were generated using one TLE from the seventh day of each month, propagated forwards and backwards to fill the entire month. They show six months of data. The error is measured in kilometers, with radial (red), in-plane (blue), and cross-track (green). Time is measured in 15 minute increments. Note that the vertical scale is very different for each figure.

The error and behavior of that error is not consistent between different GPS satellites. Figures 2,3, and 4 show that in order to get the smallest possible error in position from a TLE, each one must be analyzed separately.

A second component of the rotation matrices was found using the specific TLE and ephemeris pairs for each satellite individually.

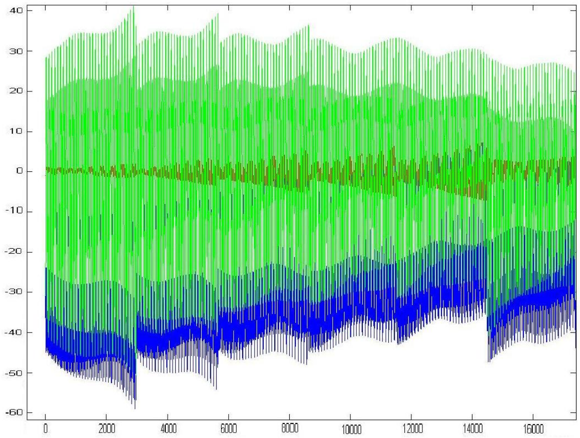

Figure 4: PRN22 error in TEME with R(Jan06)

PRN1 has the following rotation applied:

$$S_1(Jan06)^{T_{PE}}_{T_{TLE}} = \begin{pmatrix} 1 & 0 & .0015 \\ 0 & 1 & .0009 \\ -.0015 & -.0009 & 1 \end{pmatrix}$$

PRN2 has a rotation of:

$$S_2(Jan06)^{T_{PE}}_{T_{TLE}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -.0017 \\ 0 & .0017 & 1 \end{pmatrix}$$

PRN22 has a rotation of:

$$S_{22}(Jan06)^{T_{PE}}_{T_{TLE}} = \begin{pmatrix} 1 & 0 & .0015 \\ 0 & 1 & -.0008 \\ -.0015 & .0008 & 1 \end{pmatrix}$$

These rotation matrices are very stable over time, and this corresponds to a pitch and roll of 1.5 and -0.9 mrad respectively (PRN1), a roll of 1.7 mrad (PRN2), and a pitch and roll of 1.5 and $+0.8$ mrad (PRN22). The two angles change by at most ± 1.0 mrad over the course of two years. Again, the results seem insensitive to forward, backward, or combined propagation methods, which is indicative of a systematic bias in the TEME coordinate frame.

This type of rotation, one that is based on either the TLE or the orbit elements itself, is not accounted for in any published conversion method that these authors are aware of. The fact that the elements of these rotation matrices stay fairly constant over time, yet differ from TLE to TLE goes against any time-dependent and/or constant methods currently in use.

Figures 5, 6, 7 shows that when per TLE correction is included, the error of the predicted position improves greatly. The reader will also notice that the error behaves quite predictably each time a TLE is propagated out for any amount

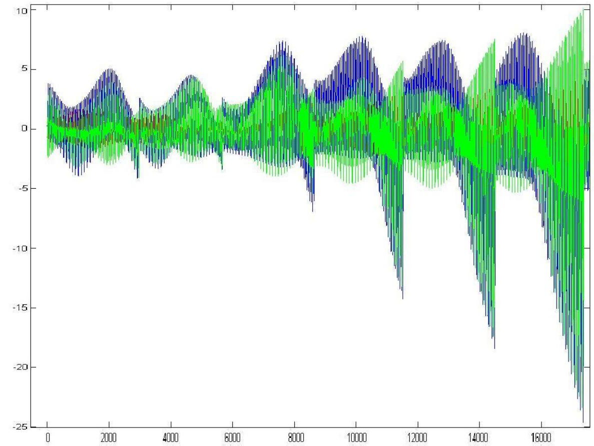

Figure 5: PRN1 error improvement, $R(Jan06)$ and $S_1(Jan06)$

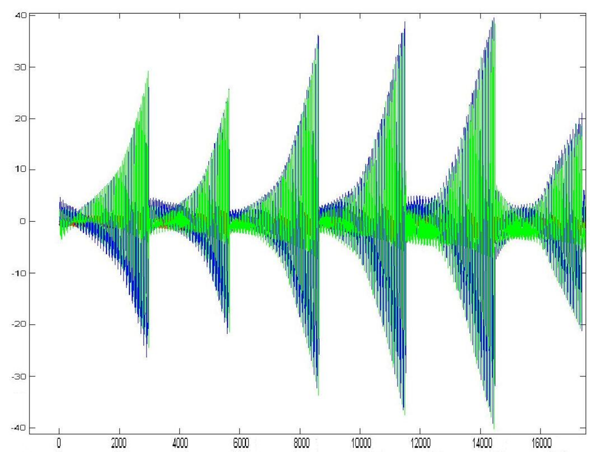

Figure 6: PRN2 error improvement, $R(Jan06)$ and $S_2(Jan06)$

of time, showing a periodic oscillation bounded by an exponential growth.

By contrasting figures 5, 6, 7 against figures 2, 3, 4, it can be seen that the error is lower in each case. Specifically the error is much lower near where the TLE was valid for a given month, one week into each month in this case. The error growth is a product of the propagation method used and appears predictable. This shows promise of being able to reduce the error of a TLE propagated out for a month to spot conjunctions farther out into the future. The sudden drops in error are at the month boundaries when a new TLE is used. Even using only these two rotations, R and S, the error estimate is much improved, even six months later.

If the same rotation is applied to the same satellite 12 months or two years in the future, the error is still improved,

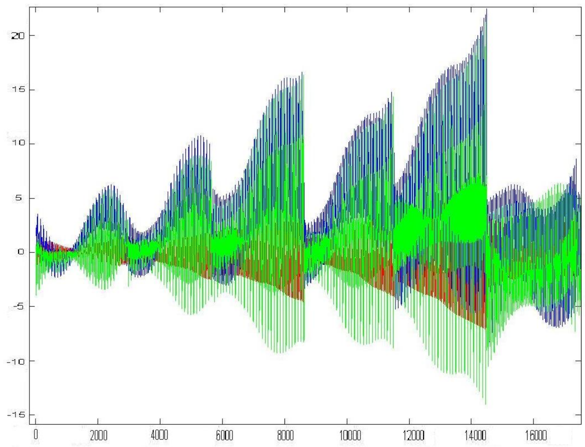

Figure 7: PRN22 error improvement, $R(Jan06)$ and $S_{22}(Jan06)$, The error in the T_{TLE} frame compared to the T_{PE} frame as shown in RIC with the consistent bias R and per TLE bias S included

but in most cases does grow slightly.

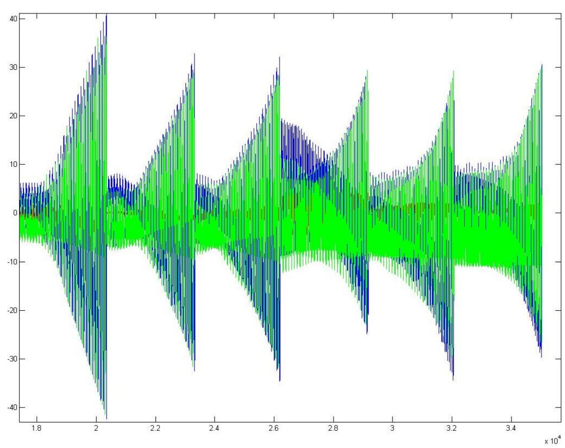

Figure 8: PRN2, 6-12 months out with $R(Jan06)$ and $S_2(Jan06)$ rotation

Figure 8 shows the error in PRN2 from July to December of 2006 after it is corrected with the rotation matrices $R(Jan06)$ and $S_2(Jan06)$, six months after the correction is generated.

Figure 9 is the error in RIC of PRN2 in January of 2008 with no correction applied at all. Figure 10 shows the error in January of 2008 with the correction from the quaternion from January 2006 applied, $R(Jan06)$ and $S_2(Jan06)$. The stability of the TEME frame bias is shown by a reduction in error of over 50%. Figure 11 shows January 2008 data corrected with a quaternion generated from data from

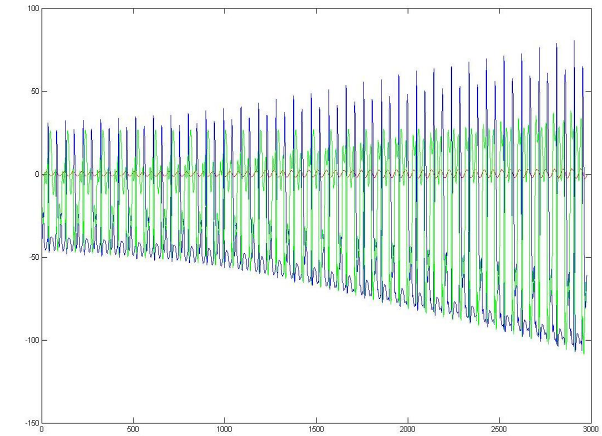

Figure 9: PRN2, Jan08 with no correction

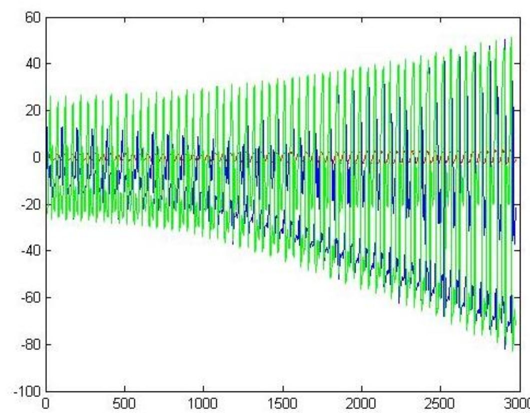

Figure 10: PRN2, Jan08 using $R(Jan06)$ and $S_2(Jan06)$ rotation corrections

the same month, $R(Jan08)$ and $S_2(Jan08)$, and represents the best correction we can apply using simple quaternion frame corrections.

It is clear from figure 12 that there is definite predictable structure in the TLE orbit data. Figure 12 shows the fast fourier transform of the RIC error for PRN2 in January of 2008. All three components show a spike that corresponds to one full orbit of the satellites. This indicated that the TLEs not only have a frame misalignment (covered by the quaternion rotation bias), but that there appears to be some orbital element mismodeling as well.

Figures 13, 14, and 15 display the improvement in error by using two different rotation corrections based on data from PRN2. The data is from January of 2008 where red corresponds to error that has not been corrected by any method, blue to error that has been corrected with

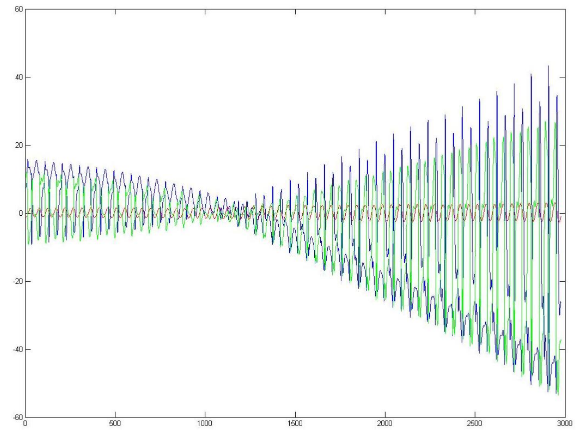

Figure 11: PRN2, Jan08 using $R(Jan08)$ and $S_2(Jan08)$ rotation corrections

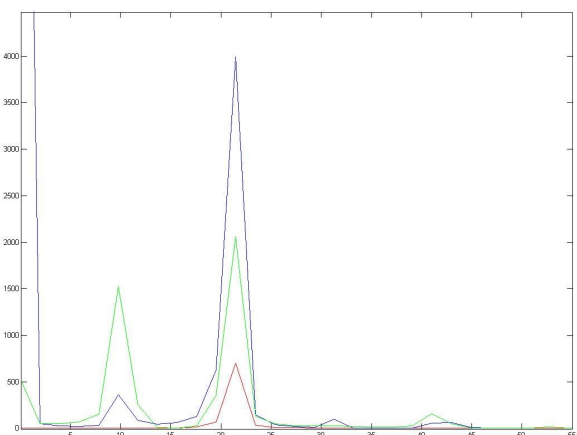

Figure 12: FFT of PRN2 error

$R(Jan06)$ and $S_2(Jan06)$, green to error that has been corrected using $R(Jan08)$ and $S_2(Jan08)$. The radial error bounds are only slightly improved by these corrections, and show maximal bounds below 3 km for a month long propagation. The in plane error, shown in figure 14, is greatly reduced when using the data from 2006. The out of track error, shown in figure 15, is improved with the 2006 rotation biases, but not as significantly as the in plane error. These figures show that our method results in an improvement when we correct the positions on per TLE basis.

Although the best fit quaternion for PRN2 changes quite a bit between 2006 and 2008, apply the rotation from 2006 still results in a much better error estimate than without this correction.

The best rotation for PRN2 in January 2008 looking at

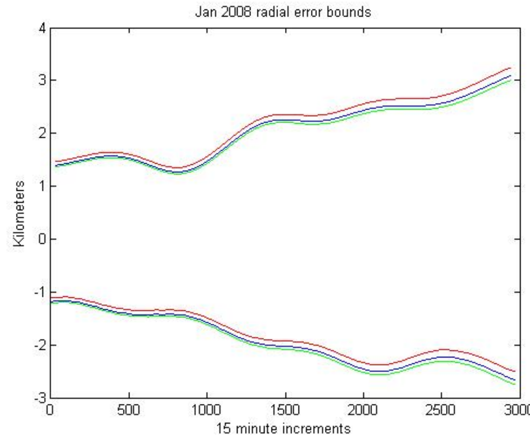

Figure 13: PRN2 radial error bounds

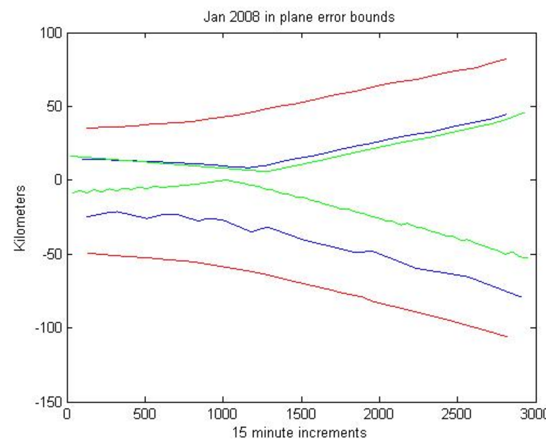

Figure 14: PRN2 in plane error bounds

only January 2008 data:

$$S_2(Jan08) = \begin{pmatrix} 1 & 0 & -.0012 \\ 0 & 1 & -.0019 \\ .0012 & .0019 & 1 \end{pmatrix}$$

versus the rotation from January 2006:

$$S_2(Jan06)^{T_{PE}}_{T_{TLE}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -.0017 \\ 0 & .0017 & 1 \end{pmatrix}$$

This corresponds to an increase in roll of 0.02 mrad, and a decrease in pitch of 1.2 mrad. While these techniques appear to be promising, we have thus far investigated only GPS satellites over the past two years. This is due to the fact that the precise ephemerides are only available starting in January of 2006.

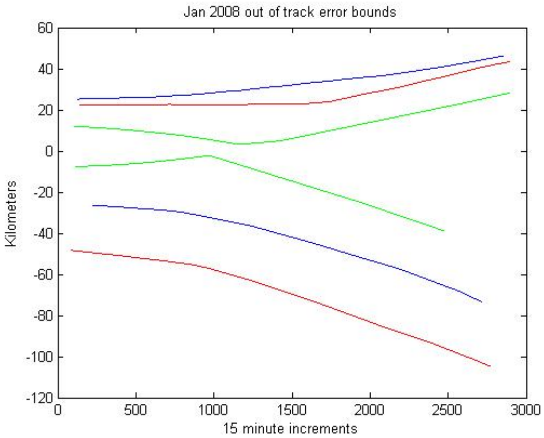

Figure 15: PRN2 out of track error bounds

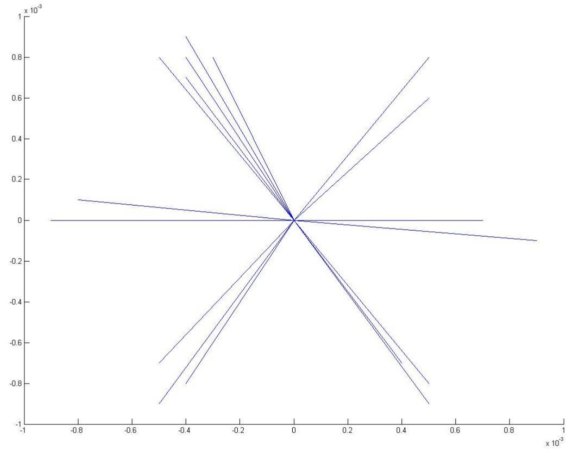

Figure 17: Jan 2006, looking down from Z-axis

3. Comparing Quaternions

The amount of precise data available for the GPS satellites allows for a very thorough analysis of TLEs, and the behavior of the TEME frame as it applies to TLEs in different orbits. We analyze 26 of the 32 GPS satellites in January of 2006, and plot the unit quaternions from T_{TLE} to T_{PE} for each satellite. The first element of the quaternion is neglected here as it will always be very close to 1 (it is the cosine of 1/2 the angle of rotation, which is very small since the two frames are so close to each other).

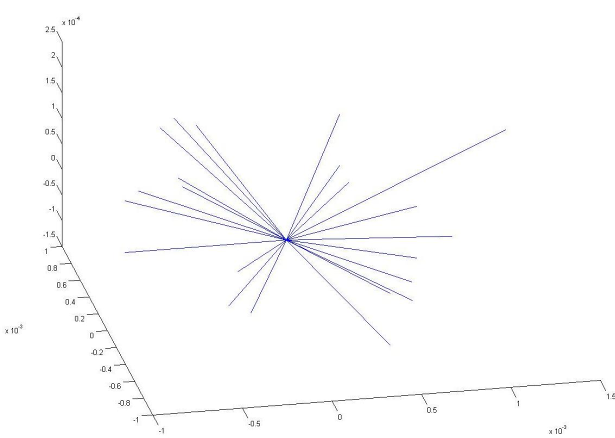

Figure 16: Jan 2006, unit quaternions for 26 of the GPS satellites

Figure 16 suggests that there is little correlation between the quaternions for each TLE, but figure 17, which is a view of the X-Y plane from above, suggests something else. It gives a good indication that the quaternions can perhaps be found based on orbit data alone. The GPS satellites are arranged in six different orbital inclinations around earth, and there are six very obvious groups of quaternions that correlate to these six different orbital inclinations.

4. Conclusions and Future Work

We have presented a method for improving the error in TLEs if more accurate position data is available for even a short period of time. This is accomplished by finding the best rotation from TLEs (T_{TLE}) to GPS precise ephemerides that have been rotated to the TEME frame (T_{PE}) using current methods. We showed that two years later in 2008, the best rotation found in 2006 is still useful in reducing position errors. Even though it is far from optimal, it still improves the error estimates greatly.

There were two parts to the bias between T_{TLE} and T_{PE}. The first is a bias between the two frames that was consistent over all of the GPS TLEs. This bias changes very slowly over time, but always stays consistent over the GPS TLEs. The second part was a rotation that was dependant on the particular GPS satellite being tested. This rotation also changes very slowly over time. We also found that there is definite structure in the residual error, which should be predictable. An analysis exploiting this structure will be a direction of future work.

We are, as of yet, unsure about the general applicability of these techniques. To date, these techniques have only been tested on GPS satellites, which lie in MEO, and all at the same inclination. However, the nature of the quaternions for each GPS satellite show that the best rotation from T_{TLE} to T_{PE} has a definite correlation to the specific characteristics of that orbit. Based on the plots of all the quaternions, they may be able to be created directly from the orbital data. This would obviate the need for more precise positioning data, and would be incredibly useful for all the orbital debris for which precise data is (and will always be) unavailable.

This work has led to some insight into the TEME coordinate frame, and at least for the GPS satellites, the TLEs suggest that there is a conversion component that differs from TLE to TLE and is very much based on the type of orbit itself. More accurate position data for other satellites is still needed to verify that this technique can be applied to other tracked space objects represented in the TEME frame.

Future research will explore the temporal correlation between TLEs and future position errors, with the hope that a filter/estimator can be derived directly from the TLEs to improve position propagation (and perhaps add a covariance to our model as well). With a covariance bound on the orbital uncertainty, conjunction analysis will produce fewer false alarms, and give the owner/operators better information as to when an evasive maneuver is actually required.

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